07/10/09

GABARITO AV1

1) $T(x[n]) = \frac{1}{5}\sum_{k=n-2}^{n+2} x[k]$

A) SISTEMA ESTÁVEL

Se $|x[n]| \leq B_x$

$$|T(x[n])| = \left|\frac{1}{5}\sum_{k=n-2}^{n+2} x[k]\right| \leq \frac{1}{5}\sum_{k=n-2}^{n+2} |x[k]| \leq B_x$$

BIBO

B) $T(x[n]) = \left(x[n-2] + x[n-1] + x[n] + \underbrace{x[n+1] + x[n+2]}\right)\frac{1}{5}$

NÃO-CAUSAL

c) $T(ax_1[n] + bx_2[n]) \overset{?}{=} aT(x_1[n]) + bT(x_2[n])$

$$\frac{1}{5}\sum_{k=n-2}^{n+2}(ax_1[k] + bx_2[k]) = \frac{1}{5}\left(a_k\sum_{k=n-2}^{n+2} x_1[k] + b\sum_{k=n-2}^{n+2} x_2[k]\right)$$

$$= a\frac{1}{5}\sum_{k=n-2}^{n+2} x_1[k] + b.\frac{1}{5}\sum_{k=n-2}^{n+2} x_2[k]$$

$$= aT(x_1[k]) + bT(x_2[k])$$ SISTEMA LINEAR

d) $y[n-n_0]$

$T(x[n-n_0])$

$$y[n] = \frac{1}{5}\sum_{k=n-2}^{n+2} x[k]$$

$$y[n-n_0] = \frac{1}{5}\sum_{k=n-n_0-2}^{n-n_0+2} x[k]$$

$$T(x[n-n_0]) = \frac{1}{5}\sum_{k=n-2}^{n+2} x[k-n_0]$$

$$= \frac{1}{5}\sum_{k=n-2-n_0}^{n+2-n_0} x[k]$$

INVARIANTE NO TEMPO

2) $h[n]$

$x[n]$

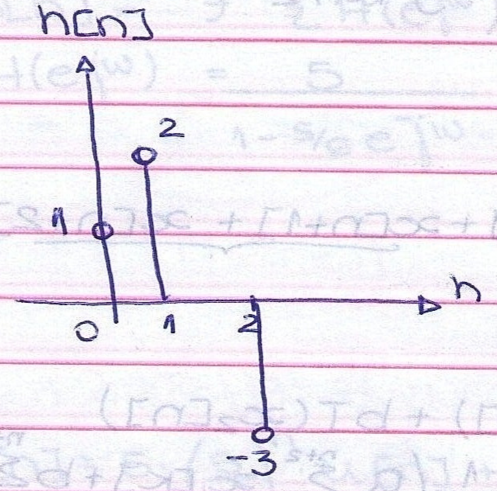
$y[n] = ?$

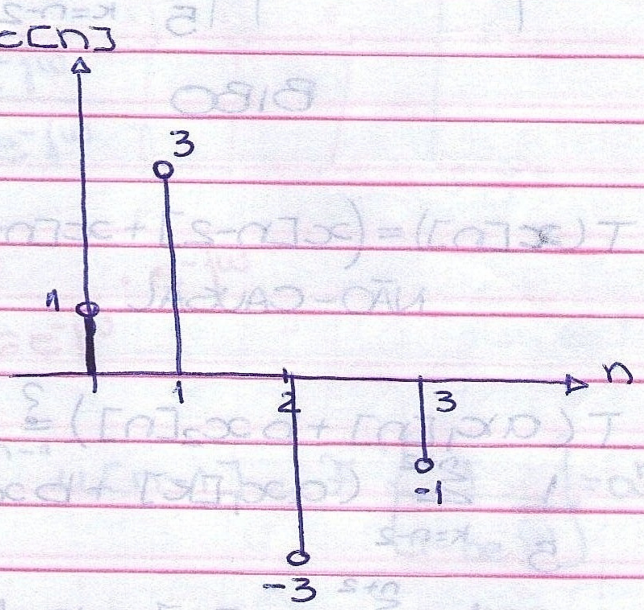
$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$

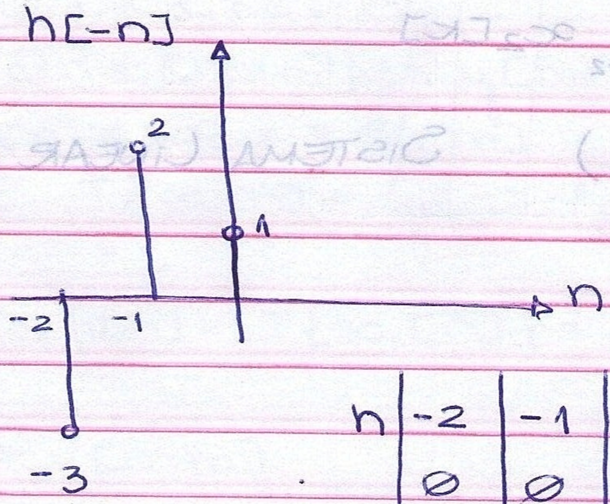
$y[n] = x[n] * h[n]$

$h[n] = \delta[n] + 2\delta[n-1] - 3\delta[n-2]$

$x[n] = \delta[n] + 3\delta[n-1] - 3\delta[n-2] - \delta[n-3]$







n	-2	-1	0	1	2	3	4	5	6
	∅	∅	1	+3	-3	-1	∅	∅	
∅	-3	2	1						
1		-3	2	1					
2			-3	2	1				
3				-3	2	1			
4					-3	2	1		

04/10/09

$x[n]$

$y[0] = 1$

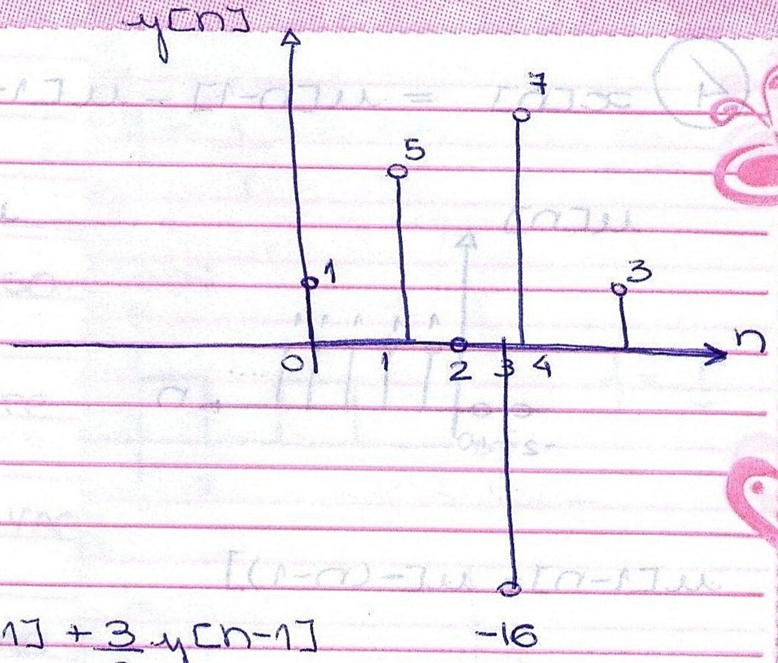
$y[1] = 2\cdot 1 + 1\cdot 3 = 5$

$y[2] = -3+6-3 = 0$

$y[3] = -9-6-1 = -16$

$y[4] = 9-2 = 7$

$y[5] = 3$



(3) $y[n] = 2x[n] + \frac{1}{8}x[n-1] + \frac{3}{8}y[n-1]$

$h[n] = y[n]\Big|_{x[n]=\delta[n]}$

$h[n] = 2\delta[n] + \frac{1}{8}\delta[n-1] + \frac{3}{8}h[n-1]$

APLICANDO T.F:

$H(e^{j\omega}) = 2 + \frac{1}{8}e^{-j\omega} + \frac{3}{8}e^{-j\omega}H(e^{j\omega})$

$H(e^{j\omega}) - \frac{3}{8}e^{-j\omega}H(e^{j\omega}) = 2 + \frac{1}{8}e^{-j\omega}$

$H(e^{j\omega})\left(1 - \frac{3}{8}e^{-j\omega}\right) = 2 + \frac{1}{8}e^{-j\omega} \Rightarrow \boxed{H(e^{j\omega}) = \frac{2 + 1/8\, e^{-j\omega}}{1 - \frac{3}{8}e^{-j\omega}}}$

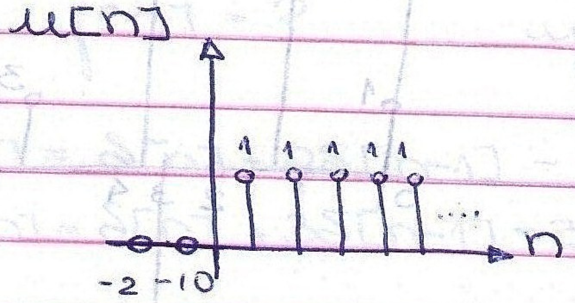
APLICANDO TRANSFORMADA INVERSA:

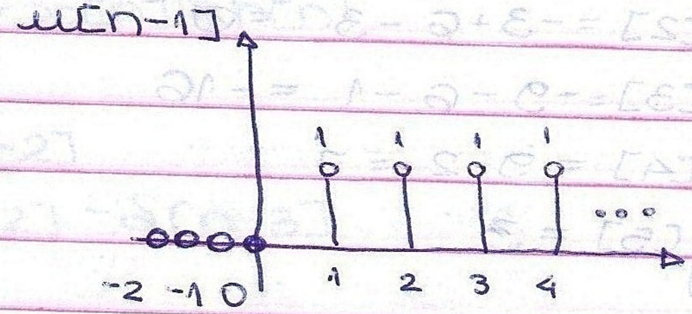
$H(e^{j\omega}) = \frac{2}{1-\frac{3}{8}e^{-j\omega}} + \frac{1/8\, e^{-j\omega}}{1-\frac{3}{8}e^{-j\omega}}$

$\mathcal{F}\{a^n u[n]\} = \frac{1}{1-ae^{-j\omega}}$

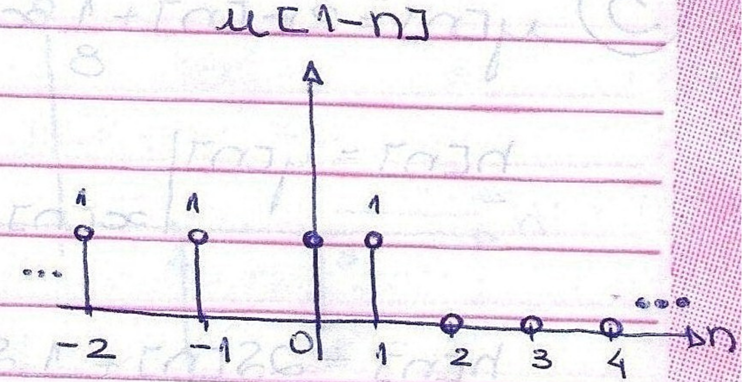
$$\boxed{h[n] = 2\cdot\left(\frac{3}{8}\right)^n u[n] + \frac{1}{8}\cdot\left(\frac{3}{8}\right)^{n-1} u[n-1]}$$

④ $x[n] = u[n-1] - u[1-n]$





$u[1-n] = u[-(n-1)]$



$n=-1$ $\quad u[1-(-1)] = u[2] = 1$

$n=0$ $\quad u[1-0] = u[1] = 1$

$n=1$ $\quad u[1-1] = u[0] = 1$

$n=2$ $\quad u[1-2] = u[-1] = 1$