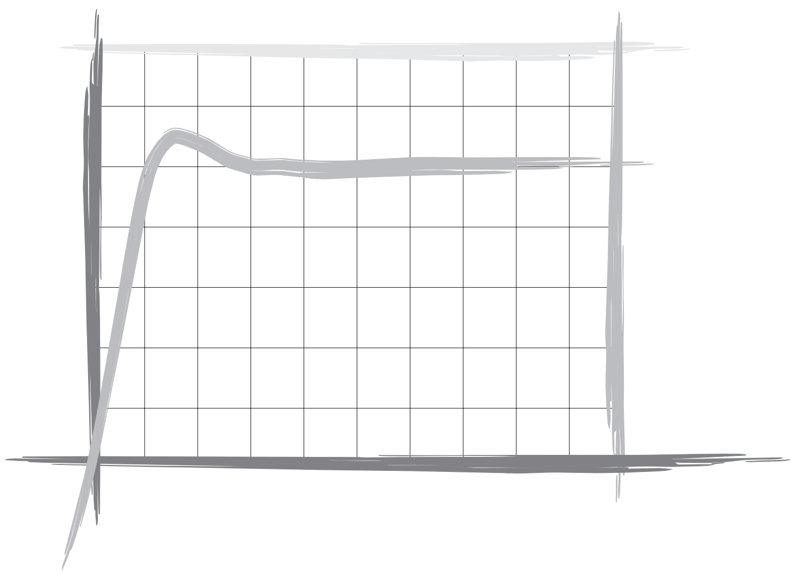
MANUAL DE SOLUÇÕES

ENGENHARIA DE CONTROLE MODERNO

4ª EDIÇÃO



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Sumário

Prefácio

Este manual apresenta a solução de todos os problemas B sem respostas. Em alguns deles, a solução apresenta informações adicionais ao resultado exigido pelo problema, que visam auxiliar o leitor.

O livro pode ser utilizado de várias maneiras, dependendo dos objetivos e da carga horária do curso a ser ministrado. Alguns exemplos:

Curso abreviado:

4 horas-aula/semana, além de
2 horas de revisão/semana

Total: 40 horas-aula, além de
20 horas de revisão

Capítulo 1
Capítulo 2*
Capítulo 3
Capítulo 4**
Capítulo 5
Capítulo 6
Capítulo 7
Capítulo 8
Capítulo 9
Capítulo 10

Curso completo:

4 horas-aula/semana, além de
2 horas de revisão/semana

Total: 56 horas-aula, além de
28 horas de revisão

Capítulo 1
Capítulo 2*
Capítulo 3
Capítulo 4
Capítulo 5
Capítulo 6
Capítulo 7
Capítulo 8
Capítulo 9
Capítulo 10
Capítulo 11
Capítulo 12

* Este capítulo poderá ser deixado de lado caso os alunos tenham conhecimento prévio dos conceitos de transformada de Laplace.

** Este capítulo poderá ser deixado de lado caso haja pouco tempo disponível.

O professor pode deixar de lado determinados itens dependendo do objetivo do curso.

Katsuhiko Ogata

CHAPTER 2

B-2-1.

(a) $$F_1(s) = \frac{s+0.4}{(s+0.4)^2+12^2} = \frac{s+0.4}{s^2+0.8s+144.16}$$

(b) $$f_2(t) = \sin\left(4t+\frac{\pi}{3}\right) = 0.5\sin 4t + 0.866\cos 4t$$

$$F_2(s) = \frac{0.5\times 4}{s^2+4^2} + \frac{0.866s}{s^2+4^2} = \frac{2+0.866s}{s^2+16}$$

B-2-2.

(a) $$f_1(t) = 3\sin(5t+45^\circ) = 2.121\sin 5t + 2.121\cos 5t$$

$$F_1(s) = \frac{2.121\times 5}{s^2+5^2} + \frac{2.121s}{s^2+5^2} = \frac{10.607+2.121s}{s^2+25}$$

(b) $$f_2(t) = 0.03 - 0.03\cos 2t$$

$$F_2(s) = \frac{0.03}{s} - \frac{0.03s}{s^2+4} = \frac{0.03s^2+0.12-0.03s^2}{s(s^2+4)}$$

$$= \frac{0.12}{s(s^2+4)}$$

B-2-3.

$$\mathcal{L}[t^2] = \frac{2}{s^3}$$

$$F(s) = \mathcal{L}[t^2e^{-at}] = \frac{2}{(s+a)^3}$$

B-2-4.

(a) $$f(t) = \sin\omega t\cdot\cos\omega t = \frac{1}{2}\sin 2\omega t$$

Hence

$$\mathcal{L}[\sin\omega t\cdot\cos\omega t] = \mathcal{L}\left[\frac{1}{2}\sin 2\omega t\right] = \frac{\omega}{s^2+4\omega^2}$$

(b) Define

$$g(t) = e^{-t} \sin 5t$$

Then

$$\mathcal{L}[g(t)] = \mathcal{L}[e^{-t} \sin 5t] = \frac{5}{(s+1)^2 + 25} = G(s)$$

Using the complex-differentiation theorem, we have

$$\mathcal{L}[t\, g(t)] = -\frac{dG(s)}{ds}$$

Hence

$$\mathcal{L}[t e^{-t} \sin 5t] = \mathcal{L}[t\, g(t)] = -\frac{d}{ds}[G(s)]$$

$$= -\frac{d}{ds}\left[\frac{5}{(s+1)^2 + 25}\right] = \frac{-10(s+1)}{[(s+1)^2 + 25]^2}$$

B-2-5.

$$f(t) = \cos 2\omega t \cdot \cos 3\omega t = \frac{1}{2}(\cos 5\omega t + \cos \omega t)$$

$$F(s) = \frac{1}{2}\left(\frac{s}{s^2 + 25\omega^2} + \frac{s}{s^2 + \omega^2}\right)$$

$$= \frac{(s^2 + 13\omega^2)s}{(s^2 + 25\omega^2)(s^2 + \omega^2)}$$

B-2-6.

$$f(t) = (t-a)\, 1(t-a)$$

$$F(s) = \frac{e^{-as}}{s^2}$$

B-2-7.

$$f(t) = t\, 1(t) - (t-T)\, 1(t-T)$$

$$F(s) = \frac{1}{s^2} - \frac{e^{-Ts}}{s^2} = \frac{1 - e^{-Ts}}{s^2}$$

B-2-8.

$$f(t) = \frac{24}{a^3} t - \frac{24}{a^2} 1\left(t - \frac{a}{2}\right) - \frac{24}{a^3}(t-a)\, 1(t-a)$$

$$F(s) = \frac{24}{a^3}\left(\frac{1}{s^2} - \frac{a e^{-\frac{a}{2}s}}{s} - \frac{e^{-as}}{s^2}\right)$$

$$\lim_{a\to 0} F(s) = \frac{24}{s^2} \lim_{a\to 0} \frac{1 - as\,e^{-\frac{a}{2}s} - e^{-as}}{a^3}$$

$$= \frac{24}{s^2} \lim_{a\to 0} \frac{\frac{d}{da}\left(1 - as\,e^{-\frac{a}{2}s} - e^{-as}\right)}{\frac{d}{da}(a^3)}$$

$$= \frac{24}{s^2} \lim_{a\to 0} \frac{-s e^{-\frac{a}{2}s} + a\frac{s^2}{2} e^{-\frac{a}{2}s} + s\,e^{-as}}{3a^2}$$

$$= \frac{8}{s^2} \lim_{a\to 0} \frac{\frac{d}{da}\left(-s e^{-\frac{a}{2}s} + a\frac{s^2}{2} e^{-\frac{a}{2}s} + s e^{-as}\right)}{\frac{d}{da}(a^2)}$$

$$= \frac{8}{s^2} \lim_{a\to 0} \frac{\frac{s^2}{2} e^{-\frac{a}{2}s} + \frac{s^2}{2} e^{-\frac{a}{2}s} - \frac{as^3}{4} e^{-\frac{a}{2}s} - s^2 e^{-as}}{2a}$$

$$= \frac{8}{s^2} \lim_{a\to 0} \frac{\frac{d}{da}\left(s^2 e^{-\frac{a}{2}s} - \frac{as^3}{4} e^{-\frac{a}{2}s} - s^2 e^{-as}\right)}{\frac{d}{da}(2a)}$$

$$= 4\left(-\frac{s}{2} - \frac{s}{4} + s\right)$$

$$= s$$

B-2-9.

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{10s}{s(s+1)} = 10$$

To verify this result, note that

$$\mathcal{L}^{-1}\left[\frac{10}{s(s+1)}\right] = (10 - 10e^{-t})\,1(t)$$

$$\lim_{t\to\infty} (10 - 10e^{-t})\,1(t) = 10$$

B-2-10.

$$f(0+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s}{(s+2)^2} = 0$$

$$\mathcal{L}[\dot{f}(t)] = sF(s) - f(0+) = sF(s)$$

Hence

$$\dot{f}(0+) = \lim_{s\to\infty} s^2 F(s) = \lim_{s\to\infty} \frac{s^2}{(s+2)^2} = 1$$

B-2-11.

$$F(s) = \frac{s+1}{s(s^2+s+1)} = \frac{1}{s} - \frac{s+0.5-0.5}{(s+0.5)^2+0.75}$$

$$= \frac{1}{s} - \frac{s+0.5}{(s+0.5)^2+0.866^2} + \frac{0.5}{0.866}\,\frac{0.866}{(s+0.5)^2+0.866^2}$$

$$f(t) = 1 - e^{-0.5t}\cos 0.866t + \frac{1}{1.732}e^{-0.5t}\sin 0.866t$$

B-2-12.

$$F(s) = \frac{5e^{-s}}{s+1}$$

Note that for a translated function $g(t-\alpha)1(t-\alpha)$, we have

$$\mathcal{L}[g(t-\alpha)\,1(t-\alpha)] = e^{-\alpha s}G(s) \qquad (\alpha \geq 0)$$

Define

$$G(s) = \frac{5}{s+1}$$

Then

$$g(t) = 5e^{-t}$$

So we have

$$\mathcal{L}[5e^{-(t-1)}1(t-1)] = e^{-s}\frac{5}{s+1}$$

or

$$f(t) = \mathcal{L}^{-1}[F(s)] = 5e^{-(t-1)}1(t-1)$$

B-2-13.

(a)

$$F_1(s) = \frac{6s+3}{s^2} = \frac{6}{s} + \frac{3}{s^2}$$

$$f_1(t) = 6 + 3t$$

(b)

$$F_2(s) = \frac{5s+2}{(s+1)(s+2)^2} = -\frac{3}{s+1} + \frac{8}{(s+2)^2} + \frac{3}{s+2}$$

$$f_2(t) = -3e^{-t} + 8te^{-2t} + 3e^{-2t}$$

B-2-14.

(a)
$$F(s) = \frac{1}{s^2(s^2+\omega^2)} = \frac{1}{\omega^2}\left(\frac{1}{s^2} - \frac{1}{s^2+\omega^2}\right)$$

$$f(t) = \frac{1}{\omega^2}\left(t - \frac{1}{\omega}\sin\omega t\right)$$

(b)
$$F_2(s) = \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} \qquad (0<\zeta<1)$$

$$= \frac{1}{s} - \frac{s+2\zeta\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$= \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_n^2-\zeta^2\omega_n^2} - \frac{\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_n^2-\zeta^2\omega_n^2}$$

Hence

$$f_2(t) = 1 - e^{-\zeta\omega_n t}\cos\omega_n\sqrt{1-\zeta^2}\,t - \frac{\zeta}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t$$

$$= 1 - e^{-\zeta\omega_n t}\left(\cos\omega_n\sqrt{1-\zeta^2}\,t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_n\sqrt{1-\zeta^2}\,t\right)$$

$f_2(t)$ can also be written as

$$f_2(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t+\phi\right)$$

where

$$\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$$

B-2-15.

$$F(s) = \frac{10(s+2)(s+4)}{(s+1)(s+3)(s+5)^2}$$

A MATLAB program to obtain the partial-fraction expansion of F(s) is given below.

```
num = conv([10  20],[1  4]);
den = conv([1  4  3],[1  10  25]);
[r,p,k] = residue(num,den)

r =

  -2.1875
   3.7500
   1.2500
   0.9375
```

```
p =

  -5.0000
  -5.0000
  -3.0000
  -1.0000

k =

  []
```

From the MATLAB output, the partial-fraction expansion of F(s) can be given as follows:

$$F(s) = \frac{-2.1875}{s+5} + \frac{3.75}{(s+5)^2} + \frac{1.25}{s+3} + \frac{0.9375}{s+1}$$

The inverse Laplace transform of F(s) is

$$f(t) = -2.1875\,e^{-5t} + 3.75\,t\,e^{-5t} + 1.25\,e^{-3t} + 0.9375\,e^{-t}$$

B-2-16.

$$F(s) = \frac{s^4 + 5s^3 + 6s^2 + 9s + 30}{s^4 + 6s^3 + 21s^2 + 46s + 30}$$

A MATLAB program to obtain the partial-fraction expansion of F(s) is given below.

```
num = [1  5  6  9  30];
den = [1  6  21  46  30];
[r,p,k] = residue(num,den)

r =

  -1.0812+ 1.7051i
  -1.0812- 1.7051i
  -0.1154
   1.2778

p =

  -1.0000+ 3.0000i
  -1.0000- 3.0000i
  -3.0000
  -1.0000

k =

   1
```

From this MATLAB output, the partial-fraction expansion of F(s) can be given as follows:

$$F(s) = \frac{-1.0812+j1.7051}{s+1-j3} + \frac{-1.0812-j1.7051}{s+1+j3} - \frac{0.1154}{s+3} + \frac{1.2778}{s+1} + 1$$

$$= 1 + \frac{-2.1624(s+1)-10.2306}{(s+1-j3)(s+1+j3)} - \frac{0.1154}{s+3} + \frac{1.2778}{s+1}$$

$$= 1 - \frac{2.1624(s+1)}{(s+1)^2+3^2} - \frac{3.4102\times 3}{(s+1)^2+3^2} - \frac{0.1154}{s+3} - \frac{1.2778}{s+1}$$

The inverse Laplace transform of F(s) becomes as follows:

$$f(t) = \delta(t) - 2.1624e^{-t}\cos 3t - 3.4102\, e^{-t}\sin 3t - 0.1154e^{-3t} + 1.2778e^{-t}$$

B-2-17. Zeros at s = -1, s = -2: z = [-1; -2]

Poles at s = 0, s = -4, s = -6: p = [0; -4; -6]

gain K = 5: K = 4

A MATLAB program to obtain B(s)/A(s) is given below.

```
z = [-1;-2];
p = [0;-4;-6];
K = 4;
[num,den] = zp2tf(z,p,K);
printsys(num,den,'s')

num/den =

   4 s^2 + 12 s + 8
   ----------------
   s^3 + 10 s^2 + 24 s
```

From the MATLAB output, we obtain

$$\frac{B(s)}{A(s)} = \frac{4s^2+12s+8}{s^3+10s^2+24s}$$

B-2-18.

$$2\ddot{x} + 7\dot{x} + 3x = 0, \quad x(0) = 3, \quad \dot{x}(0) = 0$$

$$2[s^2X(s) - sx(0) - \dot{x}(0)] + 7[sX(s) - x(0)] + 3X(s) = 0$$

$$(2s^2+7s+3)X(s) = 6s + 21$$

$$X(s) = \frac{6s+21}{2s^2+7s+3} = \frac{3s+10.5}{(s+0.5)(s+3)}$$

$$= \frac{3.6}{s+0.5} - \frac{0.6}{s+3}$$

$$x(t) = 3.6\,e^{-0.5t} - 0.6\,e^{-3t}$$

B-2-19.

$$\dot{x} + 2x = \delta(t), \qquad x(0-) = 0$$

$$sX(s) - x(0-) + 2X(s) = 1$$

$$(s+2)X(s) = 1$$

$$X(s) = \frac{1}{s+2}$$

$$x(t) = e^{-2t}\,1(t)$$

B-2-20.

$$x(t) = a\,e^{-\zeta\omega_n t}\left(\cos\omega_n\sqrt{1-\zeta^2}\,t + \frac{b+a\zeta\omega_n}{a\omega_n\sqrt{1-\zeta^2}}\sin\omega_n\sqrt{1-\zeta^2}\,t\right)$$

$$(0 \le \zeta < 1)$$

$$x(t) = a\,e^{-\omega_n t} + (b+\omega_n a)\,e^{-\omega_n t}\,t \qquad (\zeta = 1)$$

$$x(t) = \left\{-\frac{a}{2}\left(\frac{-\sqrt{\zeta^2-1}+\zeta}{\sqrt{\zeta^2-1}}\right) - \frac{b}{2\omega_n\sqrt{\zeta^2-1}}\right\} e^{-(\zeta+\sqrt{\zeta^2-1})\omega_n t}$$

$$+\left\{\frac{a}{2}\left(\frac{\sqrt{\zeta^2-1}+\zeta}{\sqrt{\zeta^2-1}}\right) + \frac{b}{2\omega_n\sqrt{\zeta^2-1}}\right\} e^{-(\zeta-\sqrt{\zeta^2-1})\omega_n t}$$

$$(\zeta > 1)$$

B-2-21. Laplace transforming both sides of the differential equation, we get

$$sX(s) - x(0) + aX(s) = A\frac{\omega}{s^2+\omega^2}$$

or

$$(s+a)X(s) = \frac{A\omega}{s^2+\omega^2} + b$$

Solving for X(s), we obtain

$$X(s) = \frac{A\omega}{(s+a)(s^2+\omega^2)} + \frac{b}{s+a}$$

$$= \frac{A\omega}{a^2+\omega^2}\left(\frac{1}{s+a} - \frac{s-a}{s^2+\omega^2}\right) + \frac{b}{s+a}$$

$$= \left(b + \frac{A\omega}{a^2+\omega^2}\right)\frac{1}{s+a} + \frac{Aa}{a^2+\omega^2}\frac{\omega}{s^2+\omega^2}$$

$$- \frac{A\omega}{a^2+\omega^2}\frac{s}{s^2+\omega^2}$$

Inverse Laplace transform of X(s) gives

$$x(t) = \mathcal{L}^{-1}[X(s)]$$

$$= \left(b + \frac{A\omega}{a^2+\omega^2}\right)e^{-at} + \frac{Aa}{a^2+\omega^2}\sin\omega t - \frac{A\omega}{a^2+\omega^2}\cos\omega t$$

$$(t \geq 0)$$

<u>B-2-22</u>.

$$\ddot{x} + 3\dot{x} + 6x = 0, \quad x(0) = 0, \quad \dot{x}(0) = 3$$

The Laplace transform of this differential equation is

$$s^2X(s) - sx(0) - \dot{x}(0) + 3[sX(s) - x(0)] + 6X(s) = 0$$

By substituting the given initial condition to the last equation, we obtain

$$s^2X(s) - 3 + 3sX(s) + 6X(s) = 0$$

from which we get

$$X(s) = \frac{3}{s^2+3s+6} = \frac{3}{(s+1.5)^2 + \left(\frac{\sqrt{15}}{2}\right)^2} = \frac{6}{\sqrt{15}}\frac{\frac{\sqrt{15}}{2}}{(s+1.5)^2 + \left(\frac{\sqrt{15}}{2}\right)^2}$$

Hence

$$x(t) = \frac{6}{\sqrt{15}}e^{-1.5t}\sin\frac{\sqrt{15}}{2}t$$

B-2-23.

$$\ddot{x} + 2\dot{x} + 10x = e^{-t}, \qquad x(0) = 0\,, \quad \dot{x}(0) = 0$$

The forcing function e^{-t} is given at $t = 0$, when the system is at rest. Taking the Laplace transform of the differential equation, we obtain

$$s^2X(s) - s\,x(0) - \dot{x}(0) + 2[sX(s) - x(0)] + 10\,X(s) = \frac{1}{s+1}$$

By substituting the given initial condition into this last equation, we get

$$s^2X(s) + 2s\,X(s) + 10\,X(s) = \frac{1}{s+1}$$

or

$$X(s) = \frac{1}{s^2+2s+10}\,\frac{1}{s+1} = \frac{1}{9}\,\frac{1}{s+1} - \frac{1}{9}\,\frac{s+1}{(s+1)^2+3^2}$$

The inverse Laplace transform of X(s) gives

$$x(t) = \frac{1}{9}e^{-t} - \frac{1}{9}e^{-t}\cos 3t \qquad (t \geq 0)$$
